Dynamic Causal Modelling for EEG/MEG: principles

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Overview

1 DCM: introduction

2 Dynamical systems theory

3 Neural states dynamics

4 Bayesian inference

5 Conclusion

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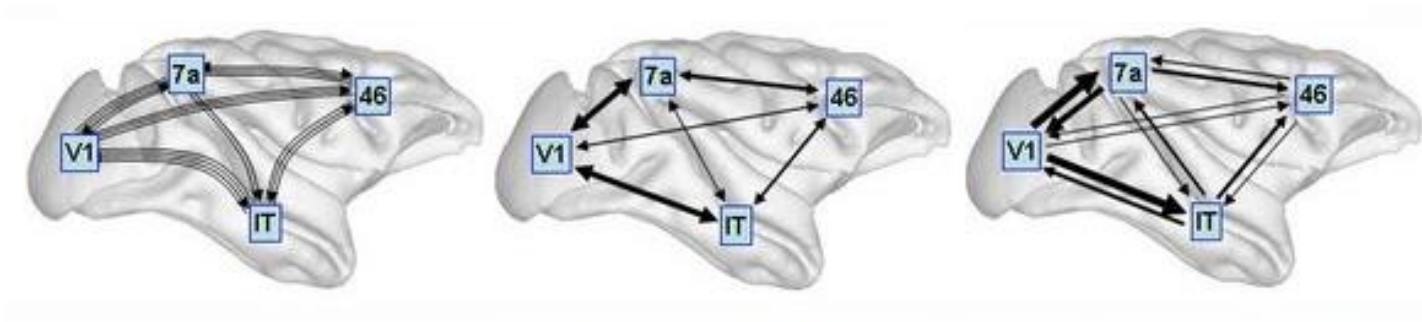
Introduction

structural, *functional* and *effective* connectivity

structural connectivity functional connectivity effective connectivity



O. Sporns 2007, *Scholarpedia*

- ***structural* connectivity**
  = presence of axonal connections

- ***functional* connectivity**
  = statistical dependencies between regional time series

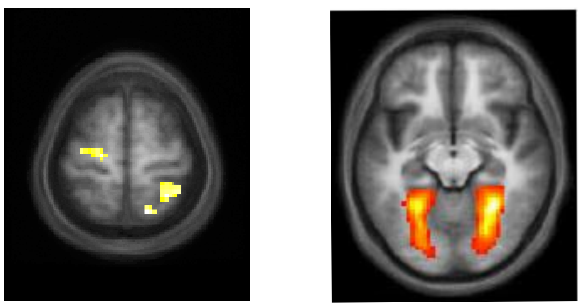
- ***effective* connectivity**
  = causal (directed) influences between neuronal populations

! connections are recruited in a *context-dependent* fashion

Introduction

from functional segregation to functional integration

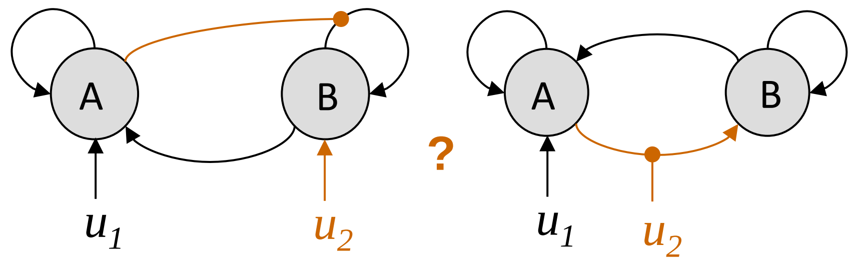
localizing brain activity:
functional segregation



u_1 $\quad$ $u_1 X u_2$

« Where, in the brain, did my experimental manipulation have an effect? »

effective connectivity analysis:
functional integration



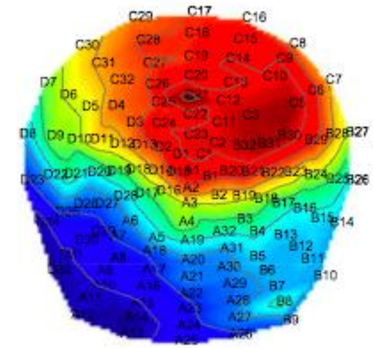
« How did my experimental manipulation propagate through the network? »

Introduction

DCM: evolution and observation mappings

Hemodynamic
observation model:
temporal convolution

Electromagnetic
observation model:
spatial convolution

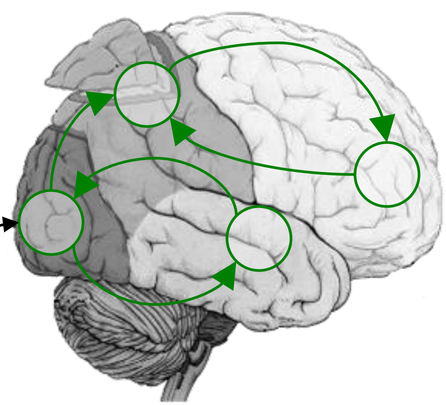


neural states dynamics

$$\dot{x} = f(x, u, \theta)$$

fMRI

EEG/MEG



inputs

- simple neuronal model
- realistic observation model

- realistic neuronal model
- simple observation model

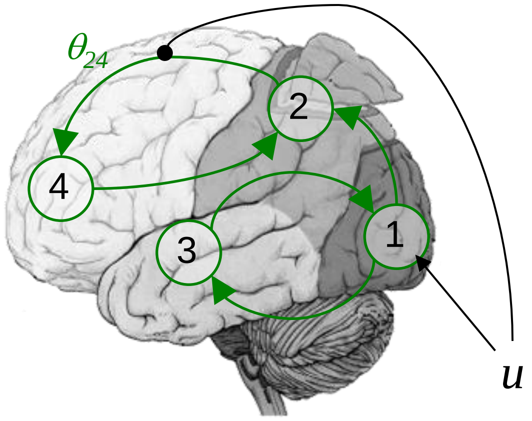
Introduction

DCM: a parametric statistical approach

- DCM: model structure

$$\begin{cases} y = g(x,\varphi) + \varepsilon \\ \dot{x} = f(x,u,\theta) \end{cases} \Rightarrow p(y|\theta,\varphi,m)$$

likelihood



- DCM: Bayesian inference

parameter estimate: $\hat{\theta} = E[\theta|y,m]$

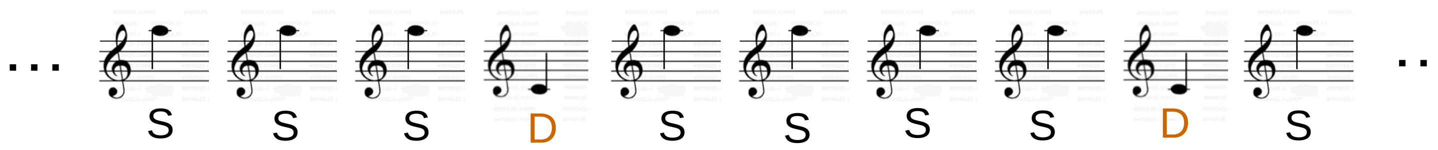
priors on parameters

model evidence: $p(y|m) = \int p(y|\theta,\varphi,m)\, p(\theta|m)\, p(\varphi|m)\, d\varphi\, d\theta$

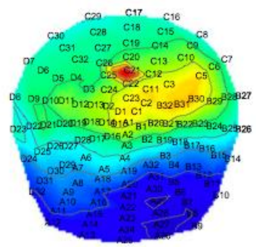
Introduction

DCM for EEG-MEG: auditory mismatch negativity

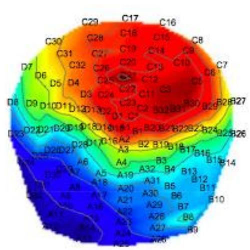
sequence of auditory stimuli



standard condition (S)

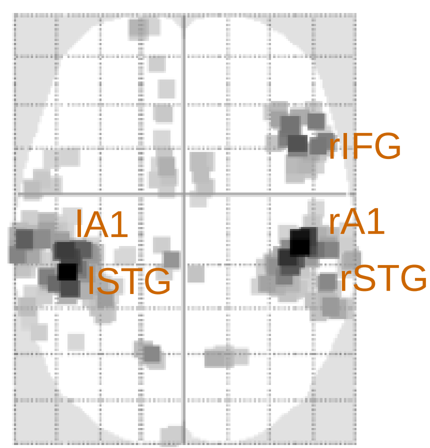


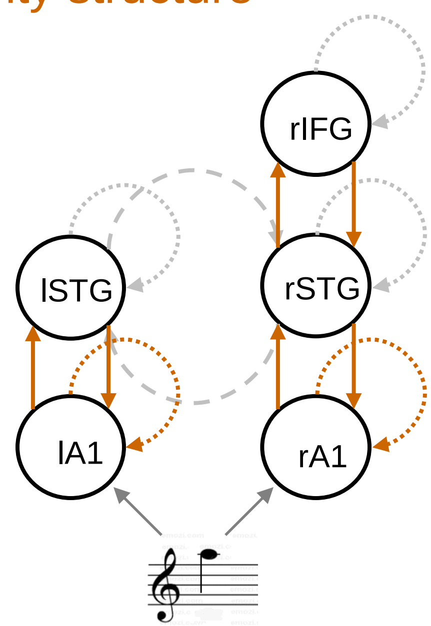
deviant condition (D)



t ˜ 200 ms

S-D: **reorganisation**
of the connectivity structure

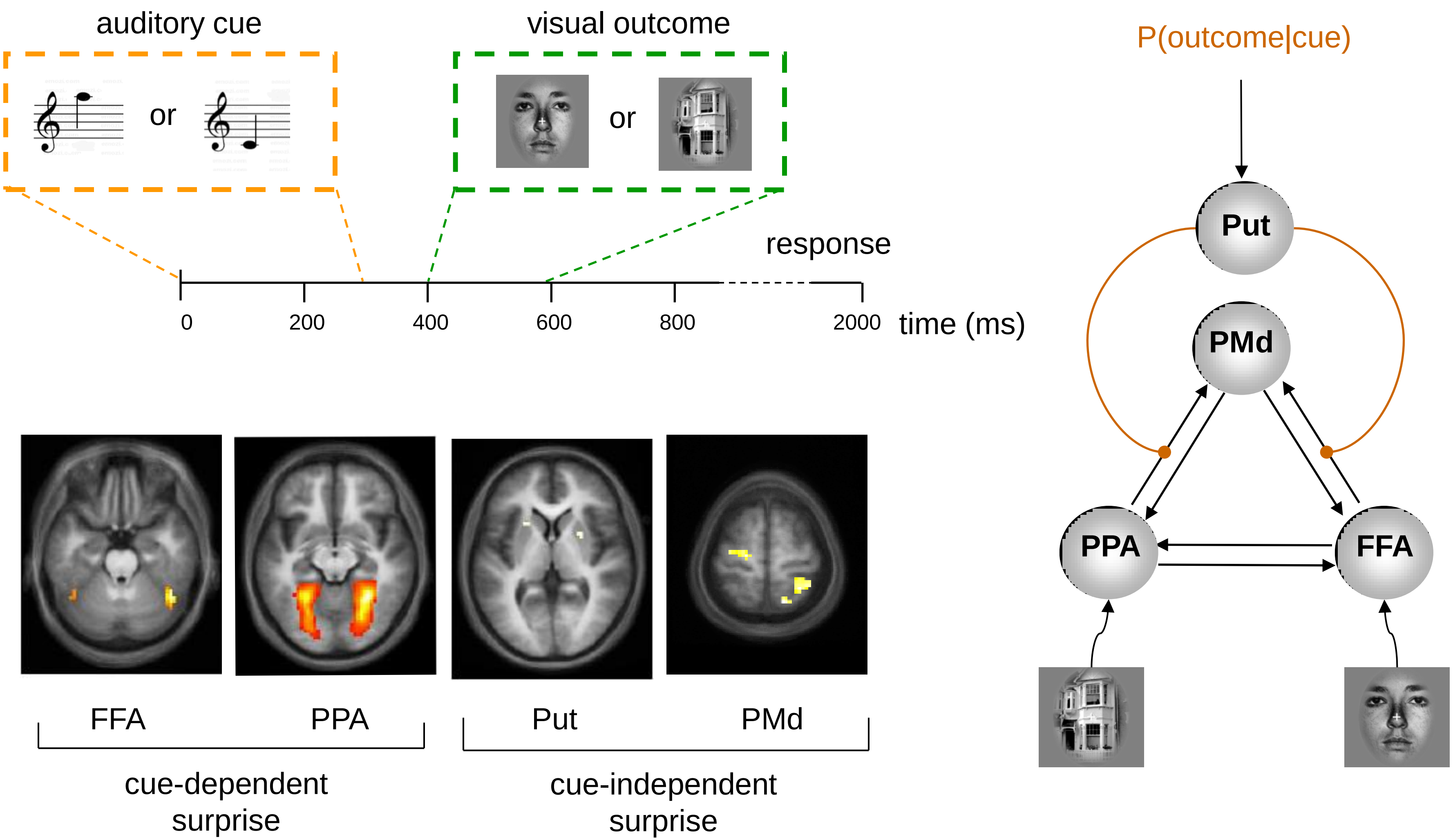




Daunizeau, Kiebel et al., Neuroimage, 2009

Introduction

DCM for fMRI: audio-visual associative learning



Den Ouden, Daunizeau et al., J. Neurosci., 2010

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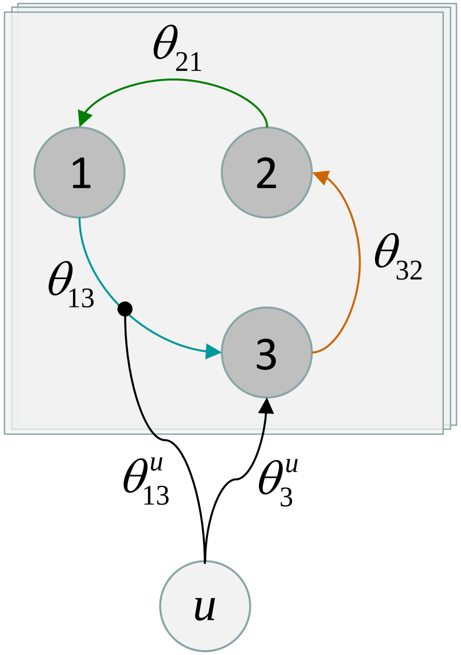
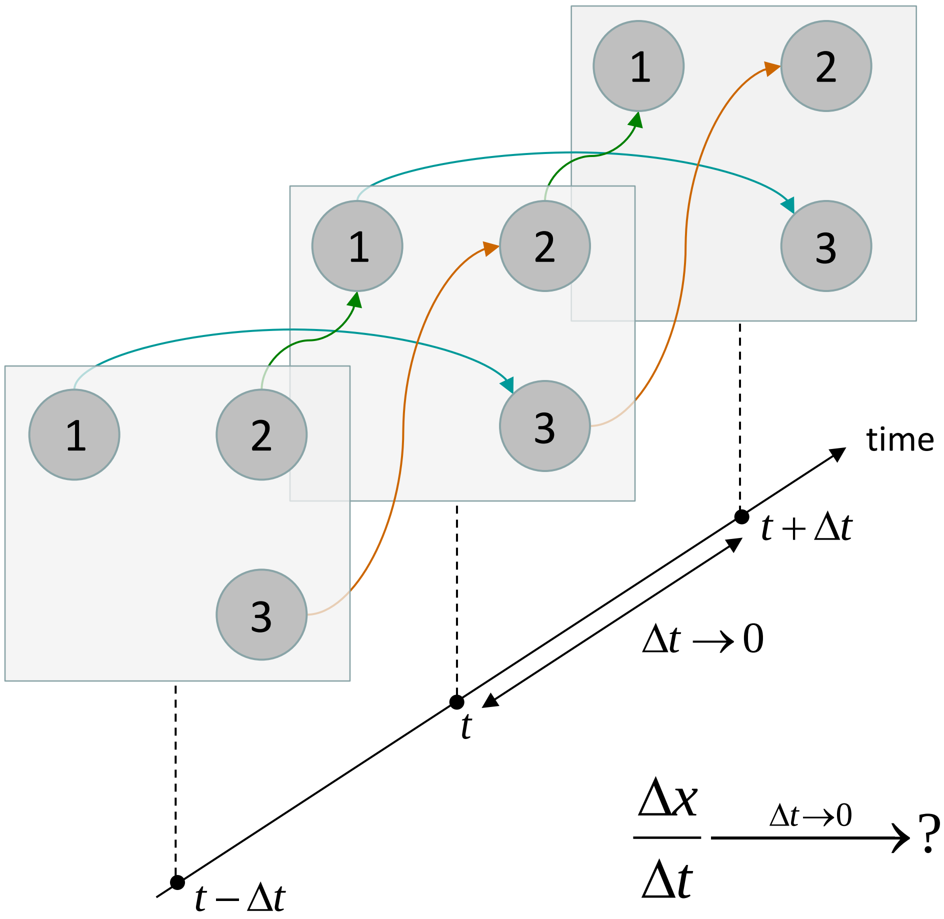
Dynamical systems theory

motivation

$$u \xrightarrow{\theta} x \xrightarrow{\varphi} y$$





$$\frac{\Delta x}{\Delta t} \xrightarrow{\Delta t \to 0} ?$$

Dynamical systems theory

exponentials

We use the following shorthand for a time derivative

$$\dot{x} = \frac{dx}{dt}$$

The exponential function $x = \exp(t)$ is invariant to differentiation. Hence

$$\dot{x} = \exp(t)$$

and

$$\dot{x} = x$$

Hence $\exp(t)$ is the solution of the above differential equation.

Dynamical systems theory

initial values and fixed points

An exponential increase ($a > 0$) or decrease ($a < 0$) from initial condition x_0

$$x = x_0 \exp(at)$$

has derivative

$$\dot{x} = a x_0 \exp(at)$$

The top equation is therefore the solution of the differential equation

$$\dot{x} = ax$$

with initial condition x_0.

The values of x for which $\dot{x} = 0$ are referred to as Fixed Points (FPs). For the above the only fixed point is at $x = 0$.

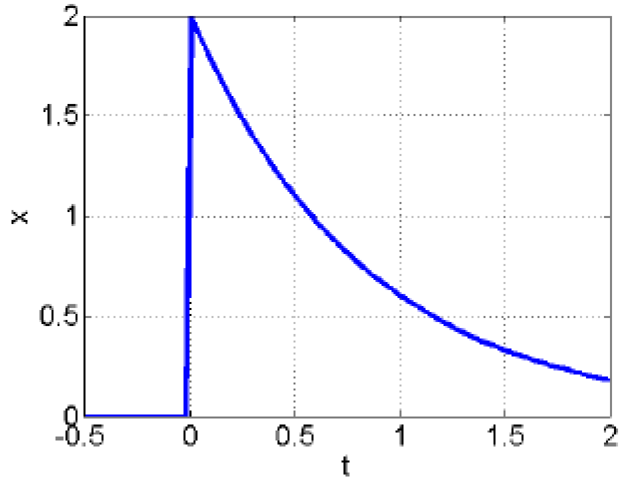
Dynamical systems theory

time constants

The figure shows

$$\dot{x} = ax$$

with $a = -1.2$ and intial value $x_0 = 2$.



The time constant is $\tau = -1/a$.

The time at which x decays to half its initial value is

$$\tau_h = \frac{1}{a} \log(1/2)$$

which equals $\tau_h = 0.58$.

Dynamical systems theory

matrix exponential

If x is a vector whose evolution is governed by a system of linear differential equations we can write

$$\dot{x} = Ax$$

where A describes the linear dependencies.

The only fixed point is at $x = 0$.

For initial conditions x_0 the above system has solution

$$x_t = \exp(At)x_0$$

where $\exp(At)$ is the matrix exponential (written expm in matlab) (Moler and Van Loan, 2003).

Dynamical systems theory

eigendecomposition of the Jacobian

The equation

$$\dot{x} = Ax$$

can be understood by representing A with an eigendecomposition, with eigenvalues λ_k and eigenvectors q_k that satisfy (Strang, p. 255)

$$A = Q\Lambda Q^{-1}$$

We can then use the identity

$$\exp(A) = Q \exp(\Lambda) Q^{-1}$$

Because Λ is diagonal, the matrix exponential simplifies to a simple exponential function over each diagonal element.

Dynamical systems theory

dynamical modes

This tells us that the original dynamics

$$\dot{x} = Ax$$

has a solution

$$x_t = \exp(At)$$

that can be represented as a linear sum of k independent dynamical modes

$$x_t = \sum_k q_k \exp(\lambda_k t)$$

where q_k and λ_k are the kth eigenvector and eigenvalue of A. For $\lambda_k > 0$ we have an unstable mode.

For $\lambda_k < 0$ we have a stable mode, and the magnitude of λ_k determines the time constant of decay to the fixed point.

The eigenvalues can also be complex. This gives rise to oscillations.

Dynamical systems theory

spirals

A spiral occurs in a two-dimensional system when both eigenvalues are a complex conjugate pair. For example (Wilson, 1999)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -16 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

has

$$\begin{aligned} \lambda_1 &= -2 + 8i \\ \lambda_2 &= -2 - 8i \end{aligned}$$

giving solutions (for initial conditions $x = [1, 1]^T$)

$$\begin{aligned} x_1(t) &= \exp(-2t)\,[\cos(8t) - 2\sin(8t)] \\ x_2(t) &= \exp(-2t)\,[\cos(8t) + 0.5\sin(8t)] \end{aligned}$$

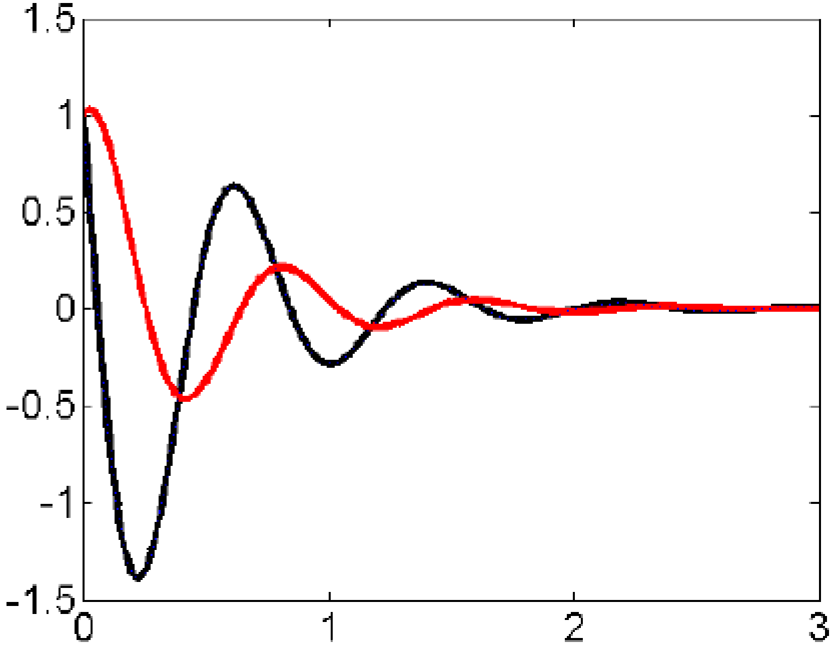
Dynamical systems theory

spirals

We plot time series solutions

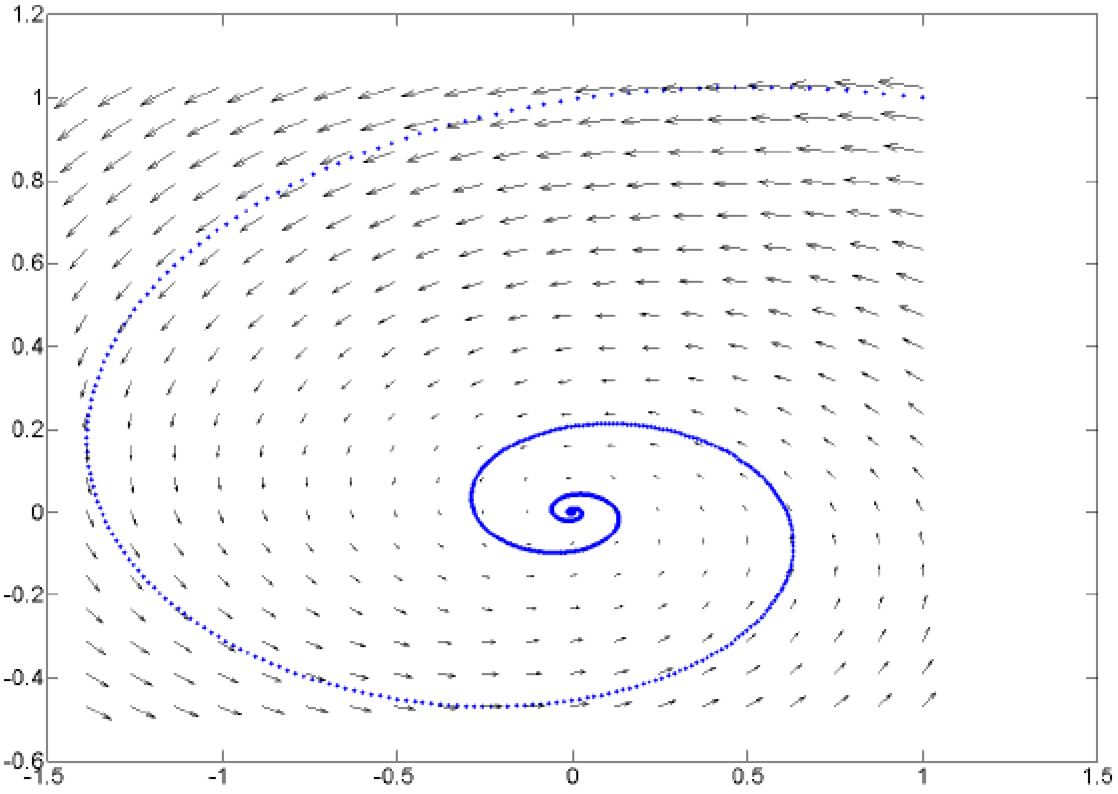
$$x_1(t) = \exp(-2t)\,(\cos(8t) - 2\sin(8t))$$
$$x_2(t) = \exp(-2t)\,(\cos(8t) + 0.5\sin(8t))$$

for x_1 (black) and x_2 (red).



Dynamical systems theory
spiral state-space

Plotting x_2 against x_1 gives the state-space representation.



Dynamical systems theory

embedding

Univariate higher order differential equations can be represented as multivariate first order DEs.

For example

$$\ddot{v} = \frac{H}{\tau} u_t - \frac{2}{\tau} \dot{v} - \frac{1}{\tau^2} v$$

can be written as

$$\begin{aligned} \dot{v} &= c \\ \dot{c} &= \frac{H}{\tau} u_t - \frac{2}{\tau} c - \frac{1}{\tau^2} v \end{aligned}$$

Dynamical systems theory

kernels and convolution

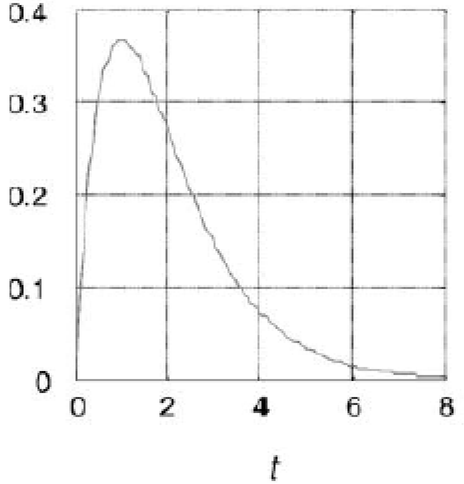
The previous differential equation has a solution given by the integral

$$v(t) = \int u(t)h(t - t')dt'$$

where

$$h(t) = \frac{H}{\tau} t \exp(-t/\tau)$$

is a kernel. In this case it is an alpha function synapse with magnitude H and time constant τ



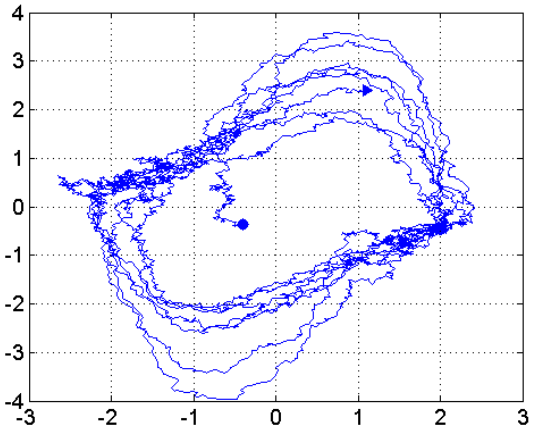
The previous integral can be written as

$$v = u \otimes h$$

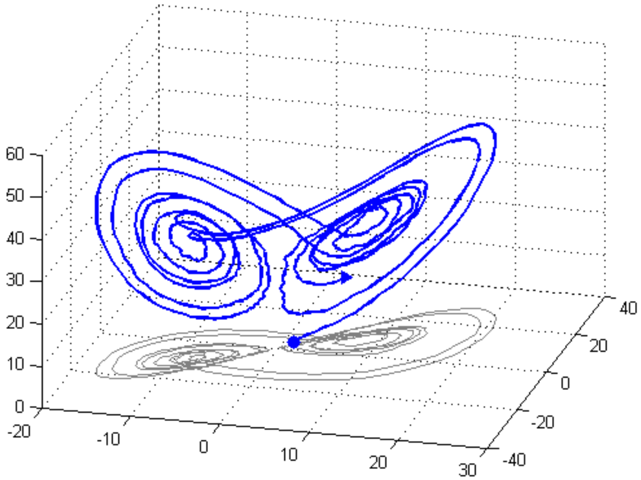
Dynamical systems theory

summary

- Motivation: modelling reciprocal influences
- Link between the integral (convolution) and differential (ODE) forms
- System stability and dynamical modes can be derived from the system's Jacobian:
    - D>0: fixed points
    - D>1: spirals
    - D>1: limit cycles (e.g., action potentials)
    - D>2: metastability (e.g., winnerless competition)



limit cycle (Vand Der Pol)



strange attractor (Lorenz)

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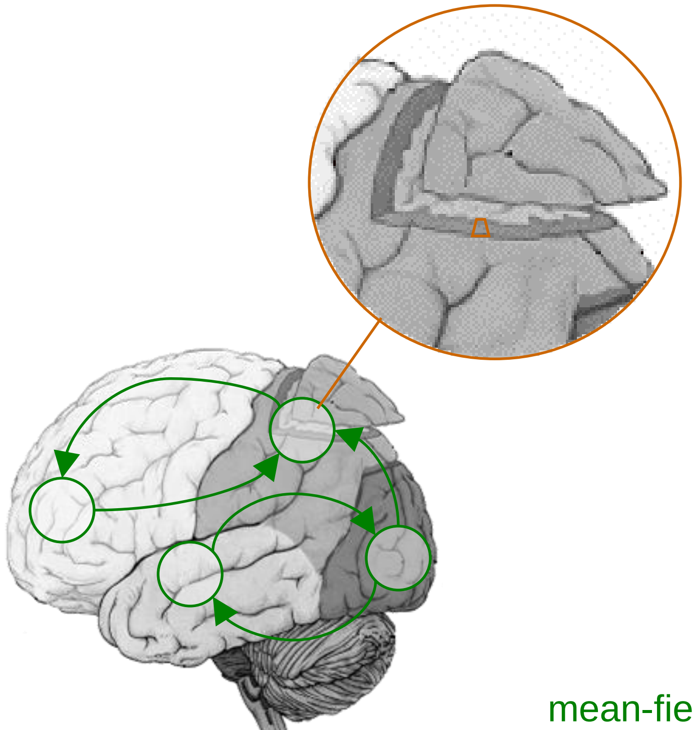
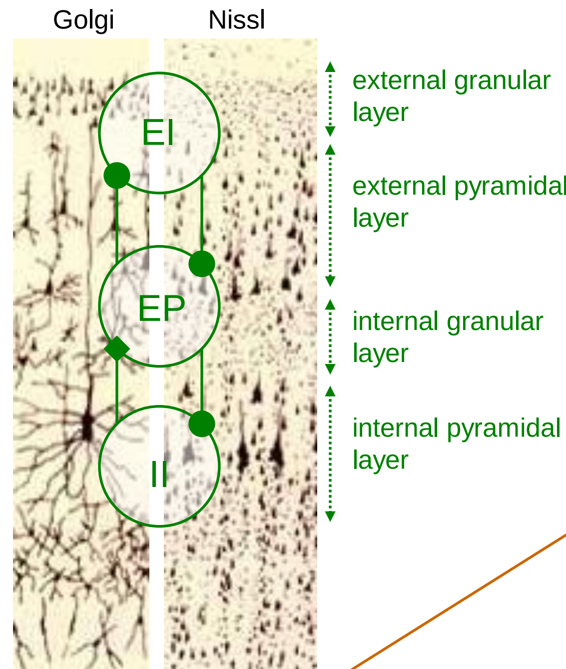
Neural ensembles dynamics

DCM for M/EEG: *systems of neural populations*

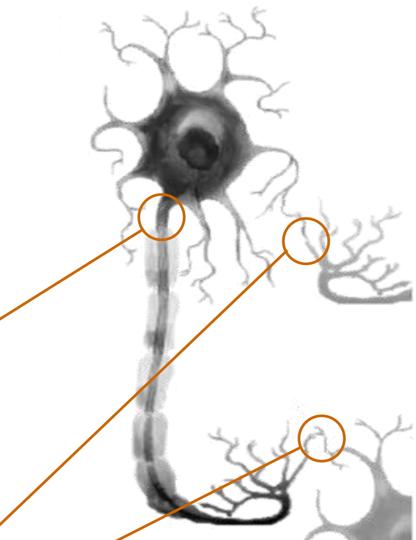
macro-scale



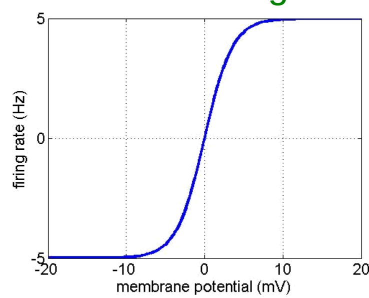
meso-scale



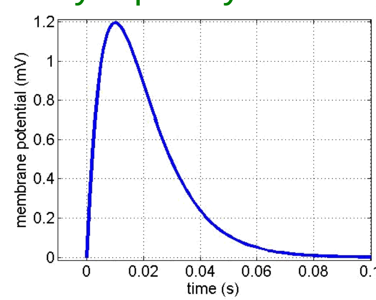
micro-scale



mean-field firing rate

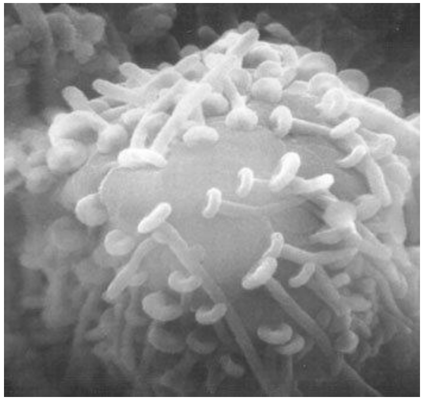


synaptic dynamics



Neural ensembles dynamics

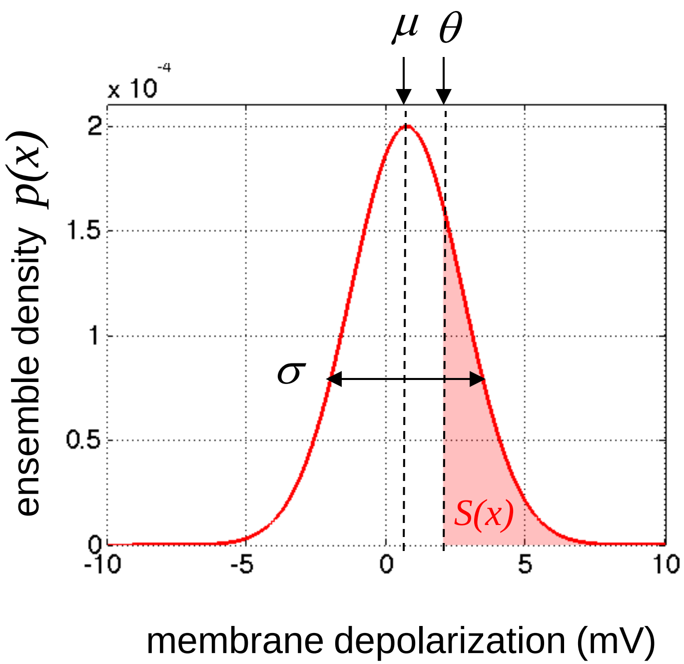
DCM for M/EEG: *from micro- to meso-scale*

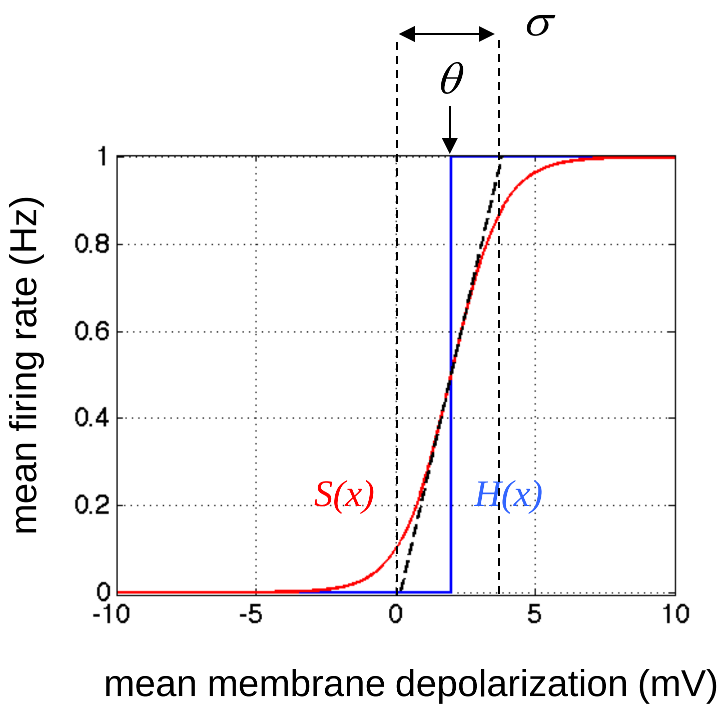


$x_j(t)$: post-synaptic potential of j^{th} neuron within its ensemble

$$\frac{1}{N-1}\sum_{j'\neq j} H\left(x_{j'}(t)-\theta\right) \xrightarrow{N\to\infty} \int H\left(x(t)-\theta\right) p\left(x(t)\right)dx$$

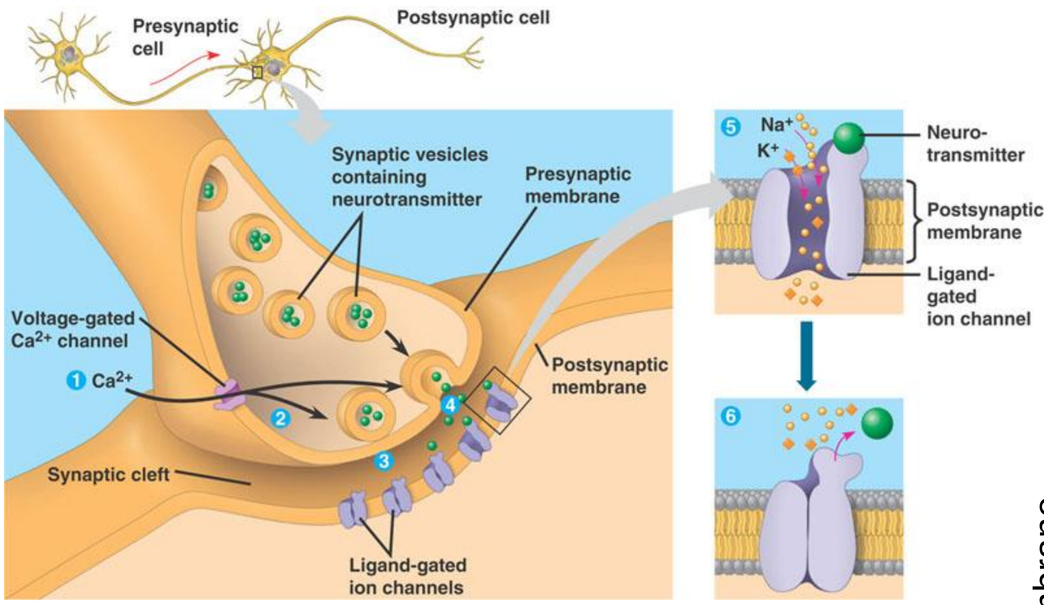
$\approx S(\mu)$ mean-field firing rate



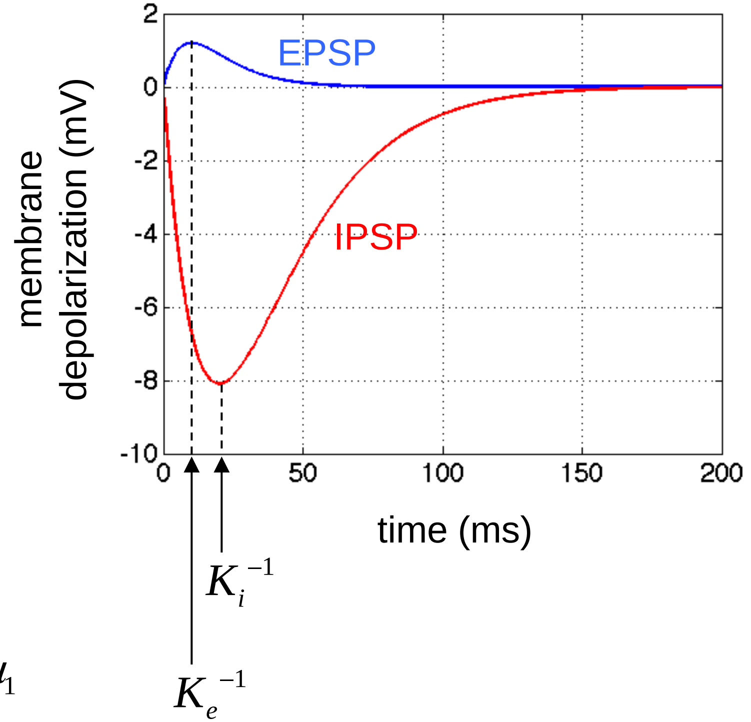


Neural ensembles dynamics

DCM for M/EEG: *synaptic dynamics*



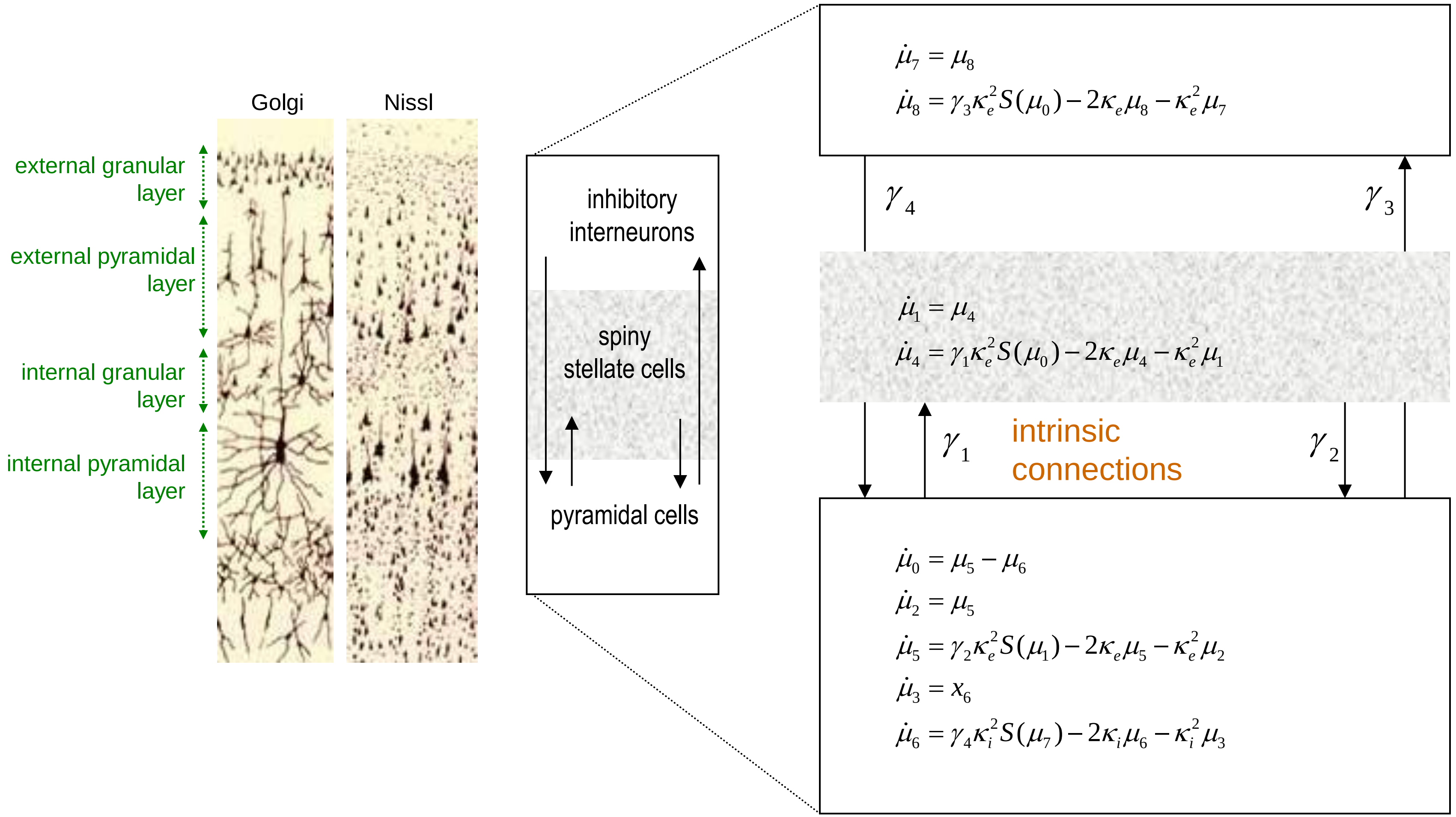
post-synaptic potential



$$\begin{cases} \dot{\mu}_1 = \mu_2 \\ \dot{\mu}_2 = \kappa_{i/e}^2 S(\bullet) - 2\kappa_{i/e}\,\mu_2 - \kappa_{i/e}^2\,\mu_1 \end{cases}$$

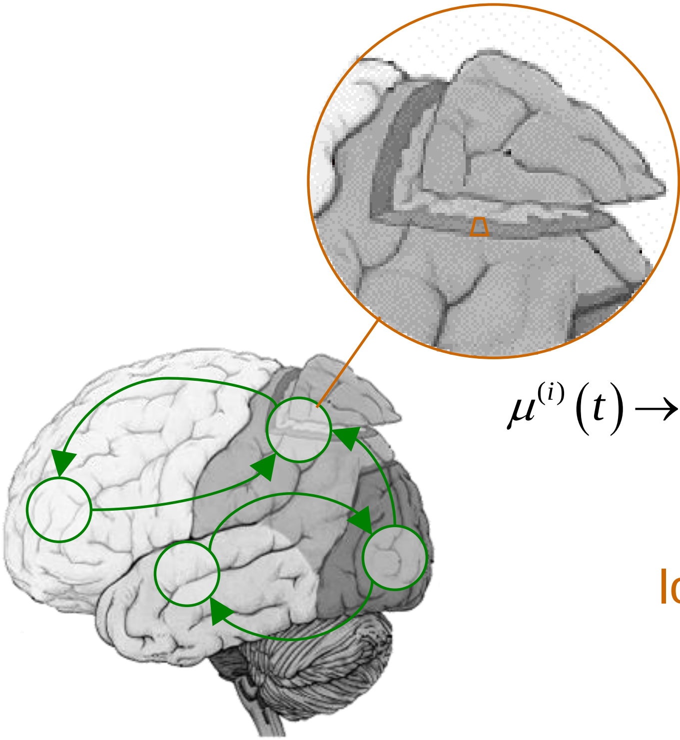
Neural ensembles dynamics

DCM for M/EEG: *intrinsic connections within the cortical column*

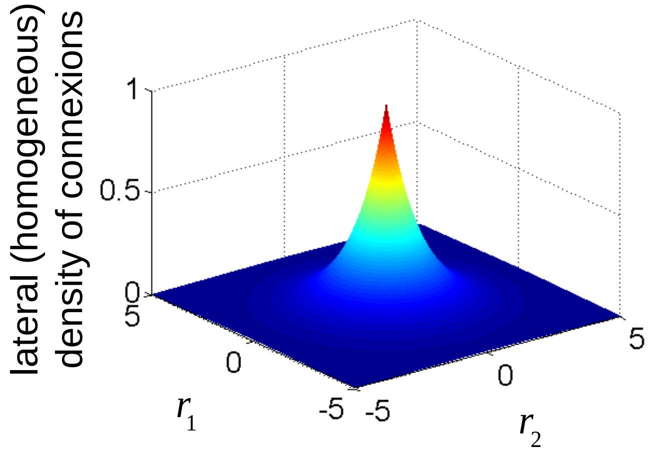


Neural ensembles dynamics

DCM for M/EEG: *from meso- to macro-scale*



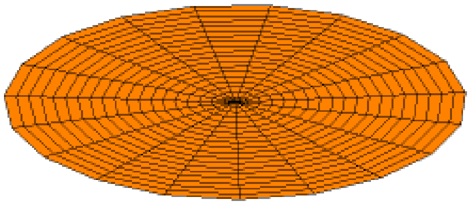
$$\mu^{(i)}(t) \rightarrow \mu^{(i)}(\mathbf{r},t)$$



local wave propagation equation (neural field):

$$\left(\frac{\partial^2}{\partial t^2} + 2\kappa\frac{\partial}{\partial t} + \kappa^2 - \frac{3}{2}c^2\nabla^2\right)\mu^{(i)}(\mathbf{r},t) \approx c\kappa\,\varsigma^{(i)}(\mathbf{r},t)$$

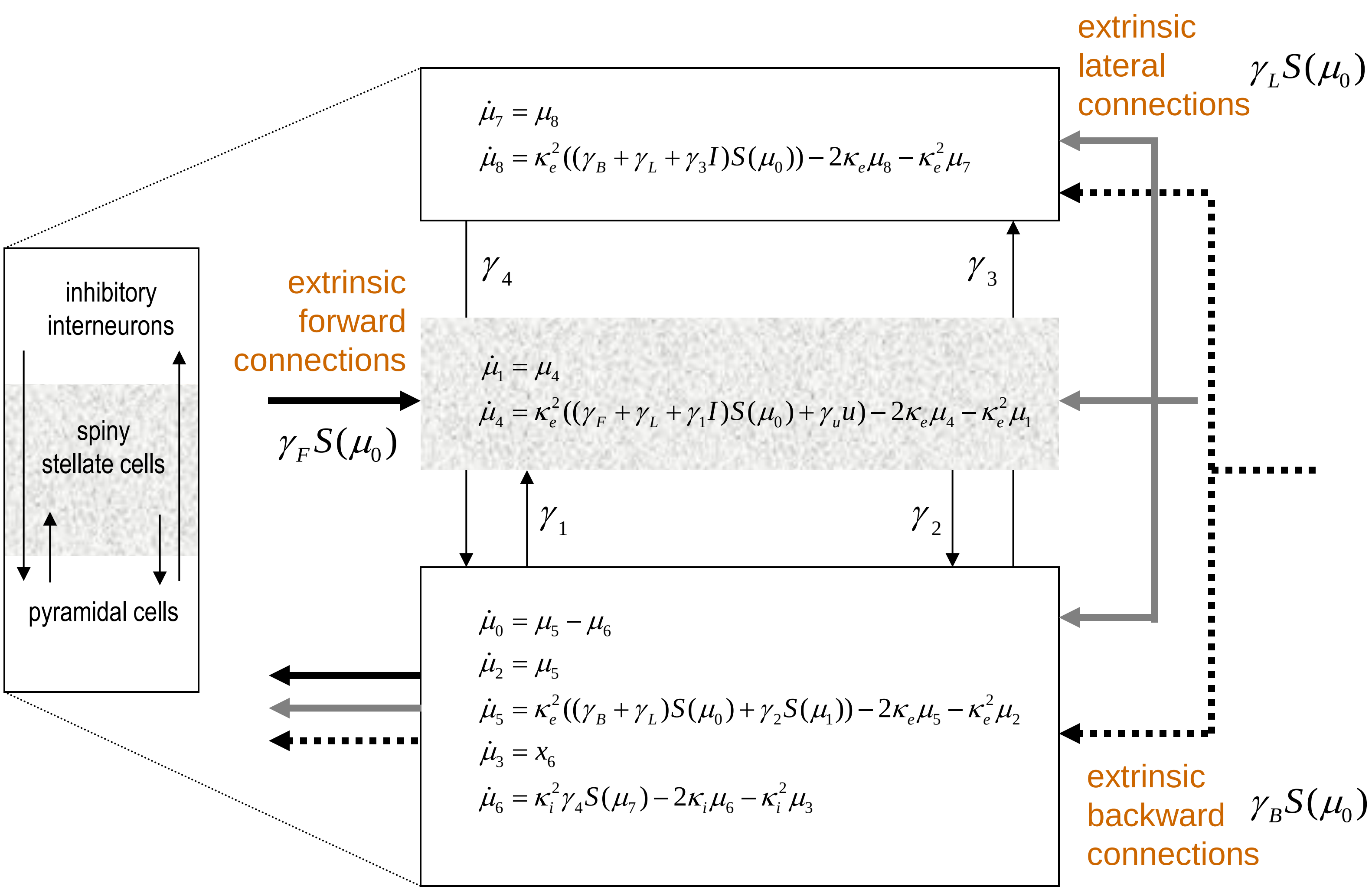
$$\varsigma^{(i)} = \sum_{i'}\gamma_{ii'}\,S\left(\mu^{(i')}\right)$$



0^{th}-order approximation: standing wave

Neural ensembles dynamics

DCM for M/EEG: *extrinsic connections between brain regions*



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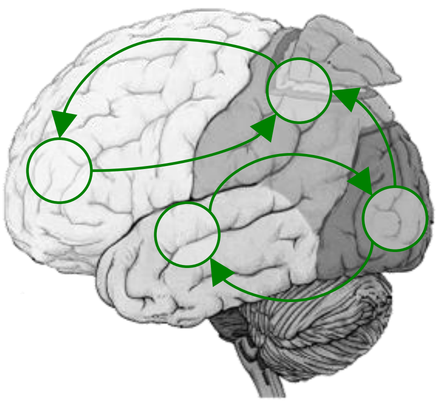
Bayesian inference

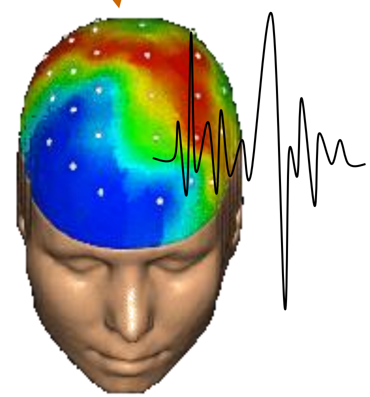
forward and inverse problems

forward problem

$$p\left(y\middle|\vartheta, m\right)$$

likelihood





posterior distribution

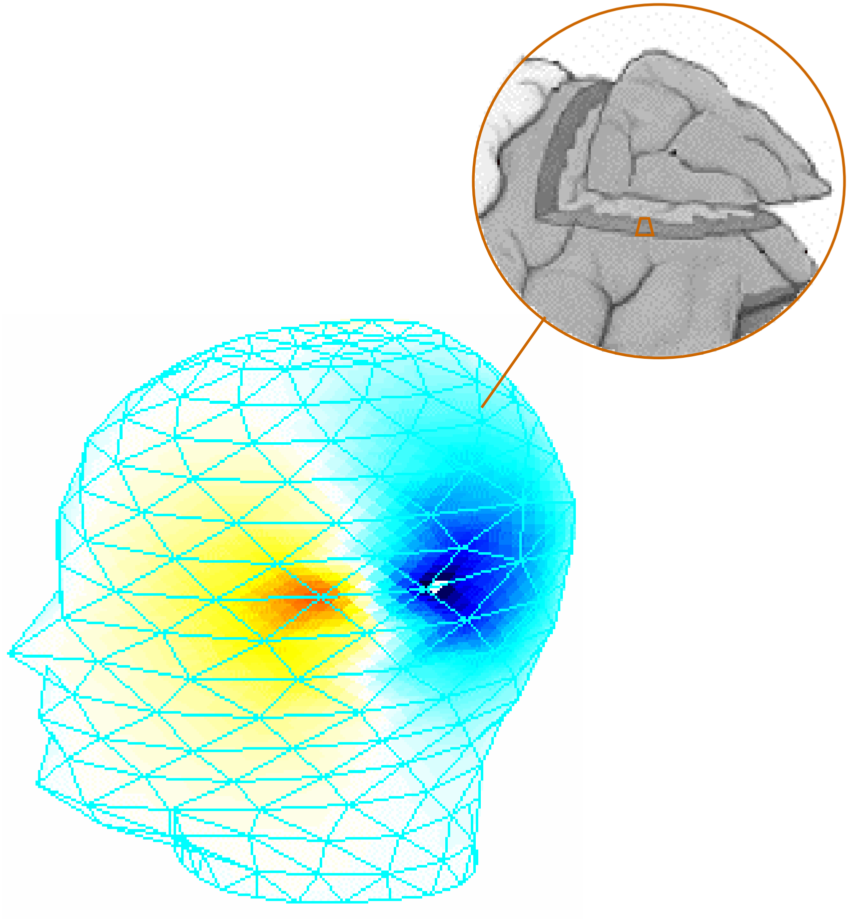
$$p\left(\vartheta\middle|y, m\right)$$

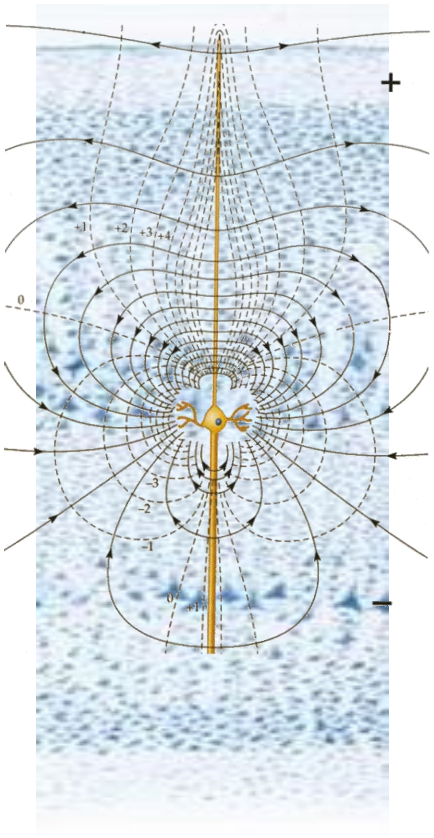
inverse problem

Bayesian inference

the electromagnetic forward problem

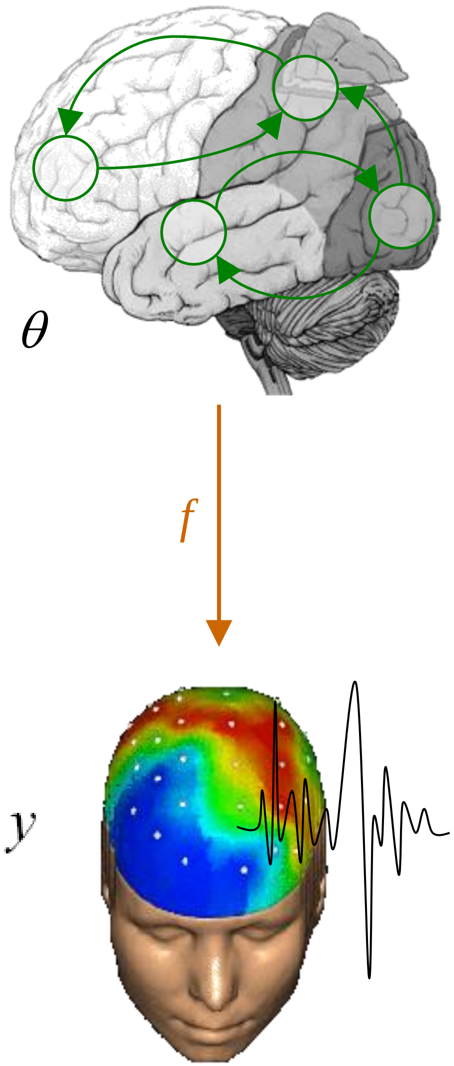
$$\mathbf{y}(t) = \sum_i \mathbf{L}^{(i)} \mathbf{w}_0^{(i)} \sum_j \beta_j \mu^{(ij)}(t) + \varepsilon(t)$$





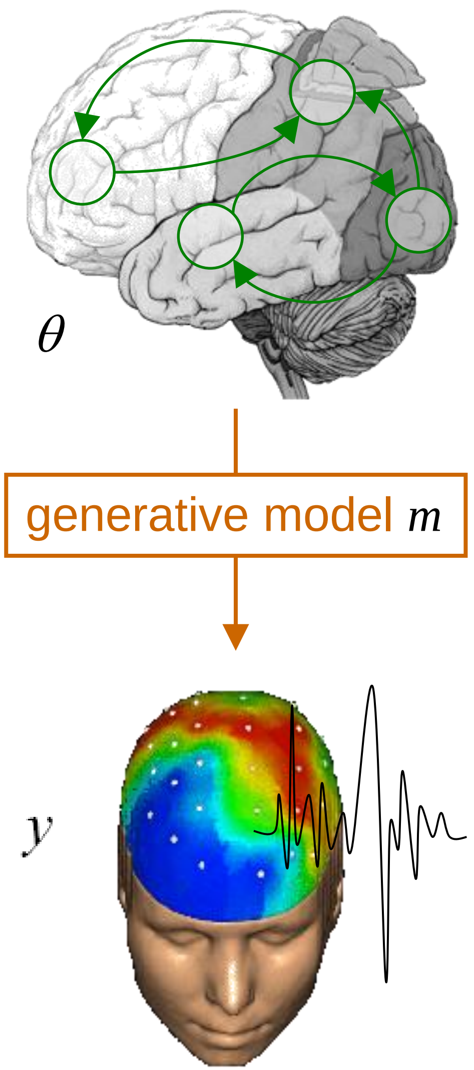
Bayesian paradigm

deriving the likelihood function



Bayesian paradigm

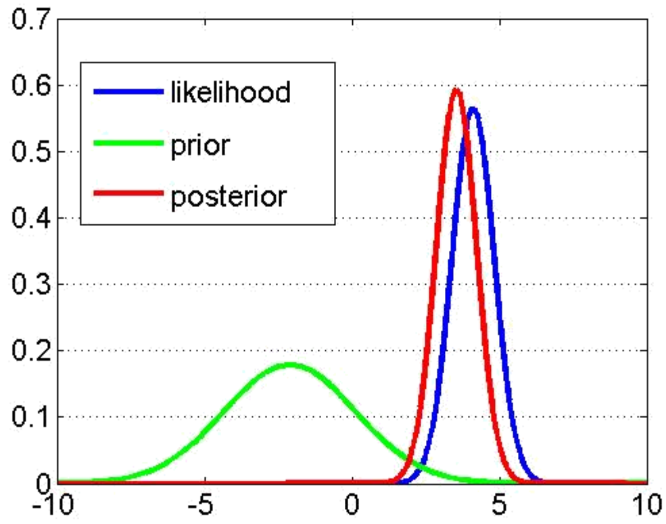
likelihood, priors and the model evidence



Likelihood: $p(y|\theta, m)$

Prior: $p(\theta|m)$

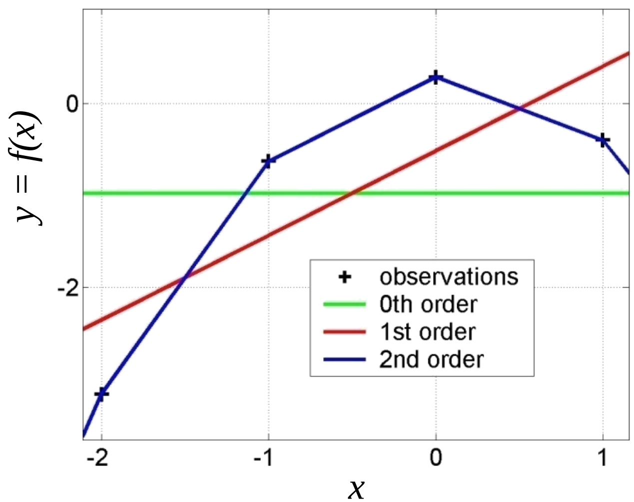
Bayes rule: $$p(\theta|y, m) = \frac{p(y|\theta, m)\,p(\theta|m)}{p(y|m)}$$

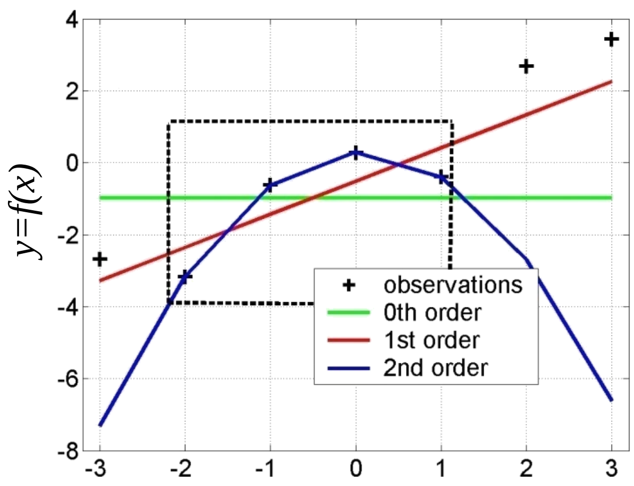


Bayesian inference

model comparison

Principle of parsimony :
« plurality should not be assumed without necessity »

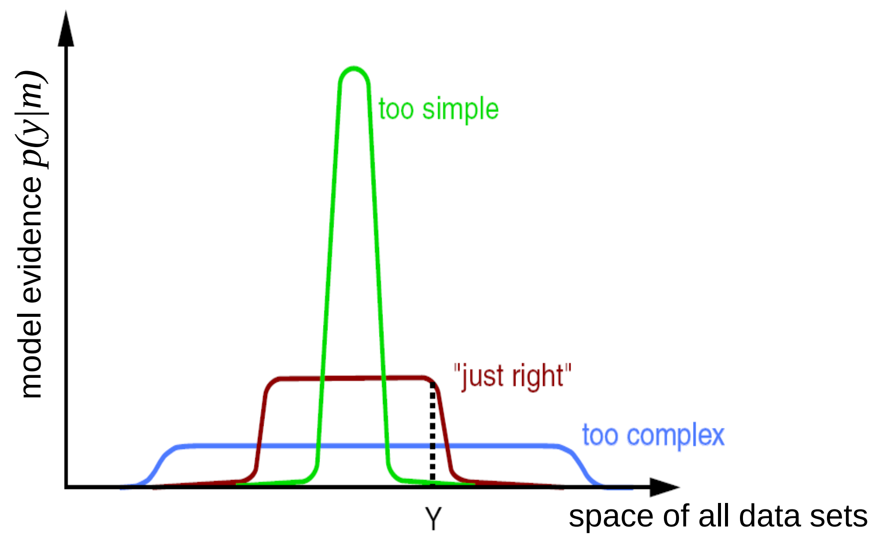




Model evidence:

$$p\left(y|m\right)=\int p\left(y|\vartheta,m\right)p\left(\vartheta|m\right)d\vartheta$$

"Occam's razor" :

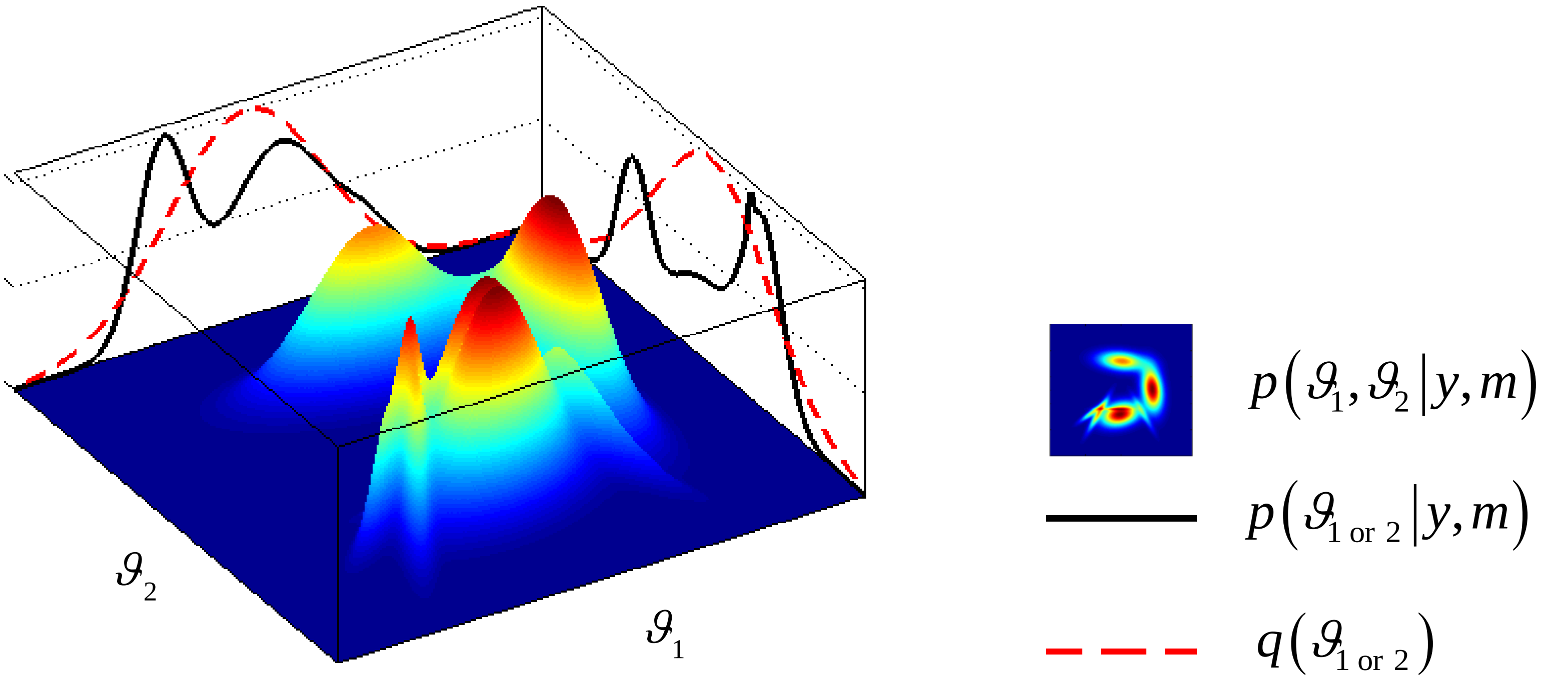


Bayesian inference

the variational Bayesian approach

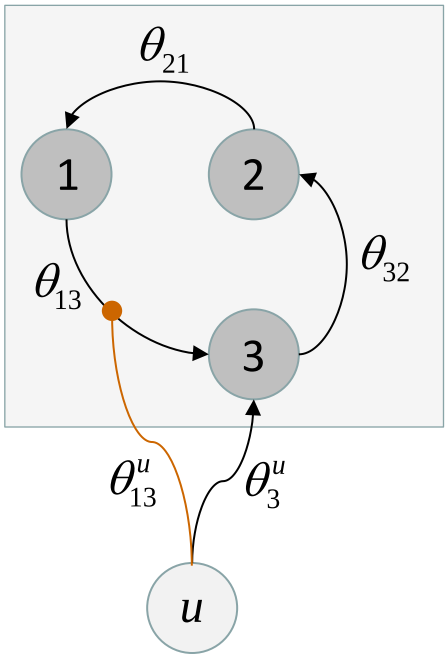
$$\ln p(y|m) = \underbrace{\left\langle \ln p(\vartheta, y|m) \right\rangle_q + S(q)}_{\text{free energy : functional of } q} + D_{KL}\left(q(\vartheta); p(\vartheta|y,m)\right)$$

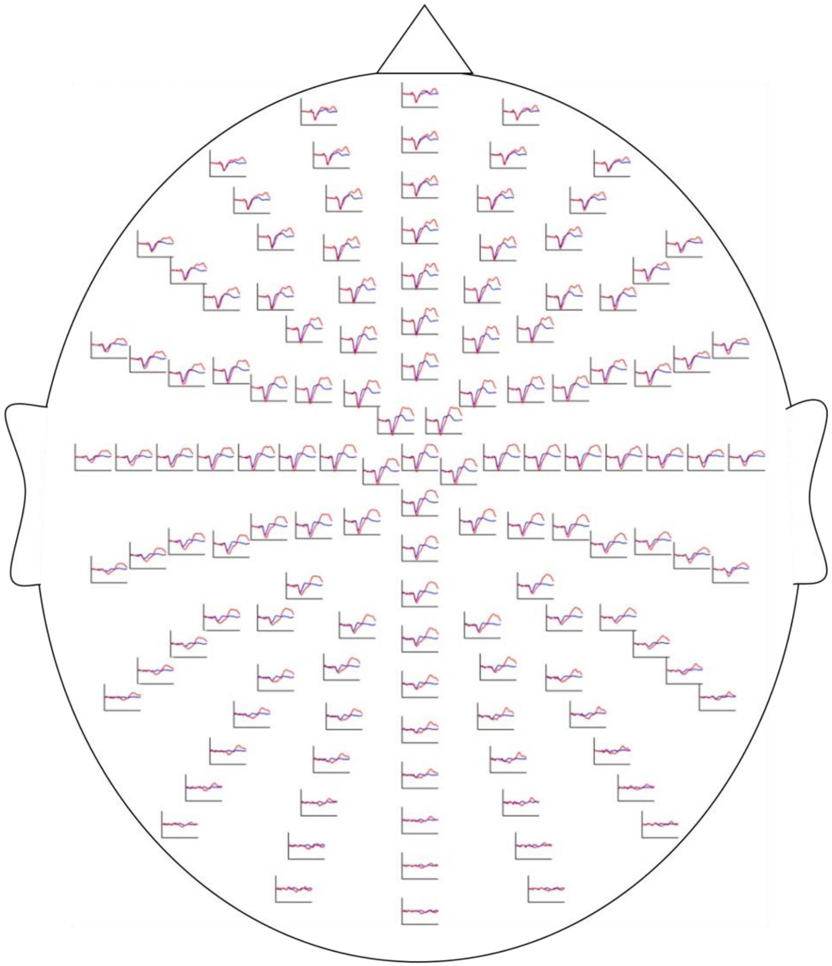
mean-field: approximate marginal posterior distributions: $\left\{ q(\vartheta_1), q(\vartheta_2) \right\}$



Bayesian inference

DCM: key model parameters





$(\theta_{21}, \theta_{32}, \theta_{13})$ state-state coupling

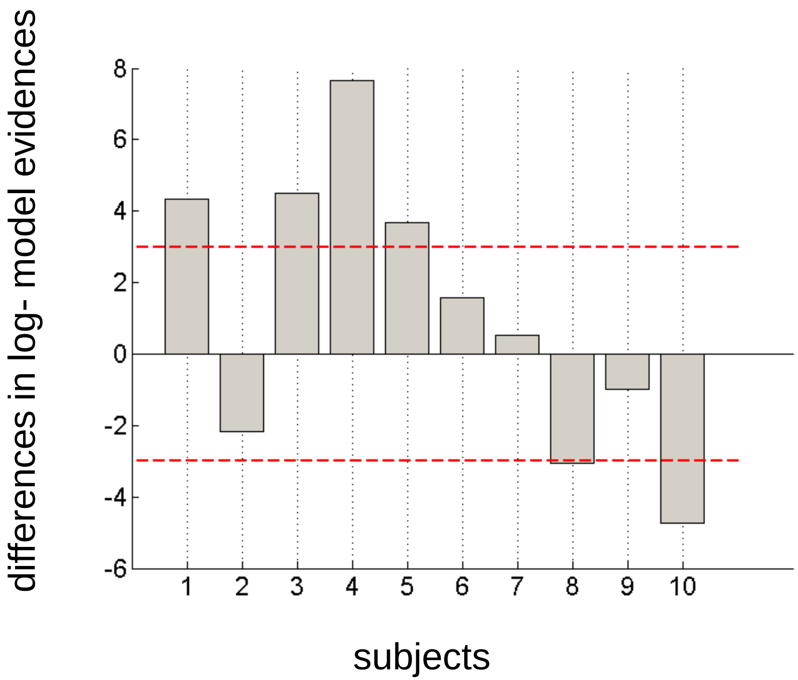
θ_{3}^{u} input-state coupling

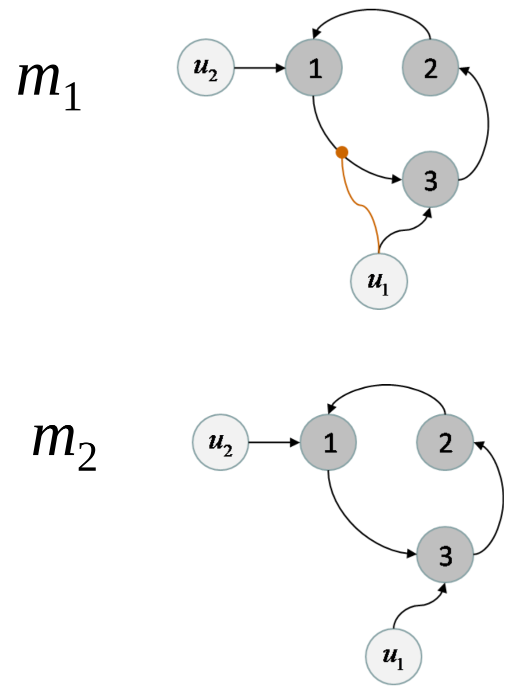
θ_{13}^{u} input-dependent modulatory effect

Bayesian inference

model comparison for group studies

$$\ln p\left(y|m_1\right) - \ln p\left(y|m_2\right)$$





fixed effect — assume all subjects correspond to the same model

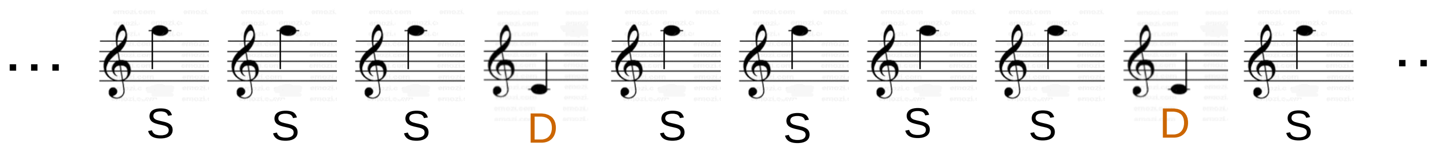
random effect — assume different subjects might correspond to different models

Overview

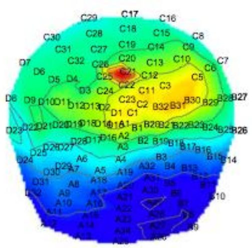
Conclusion

back to the auditory mismatch negativity

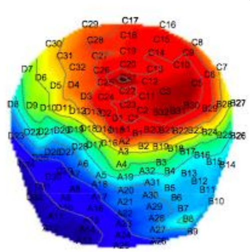
sequence of auditory stimuli



standard condition (S)

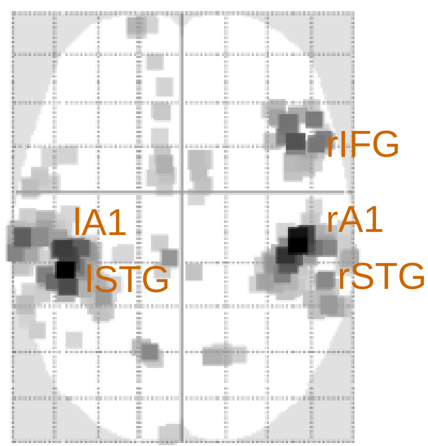


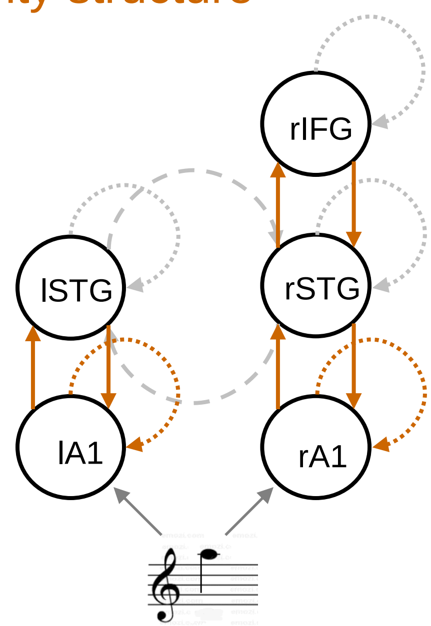
deviant condition (D)



t ~ 200 ms

S-D: **reorganisation**
of the connectivity structure

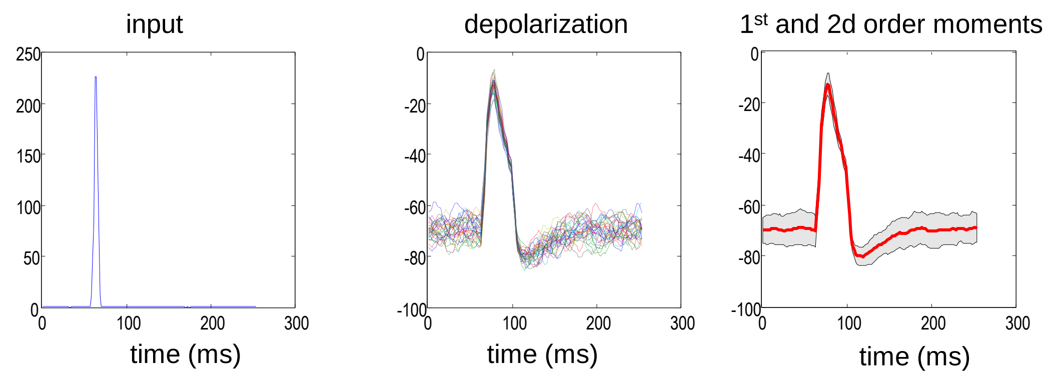




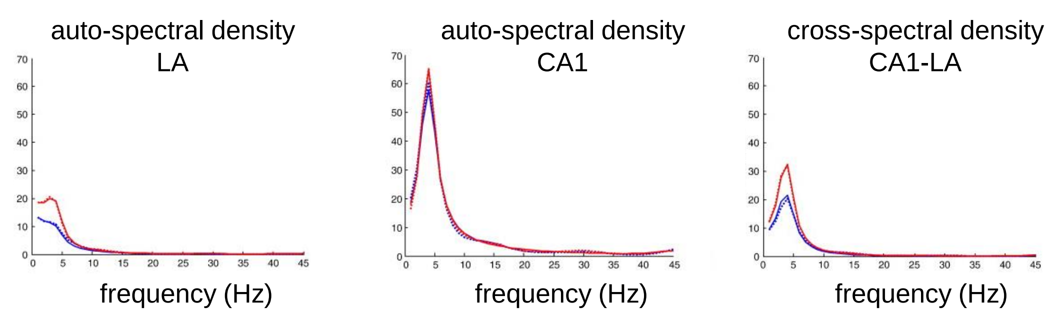
Conclusion

DCM for EEG/MEG: variants

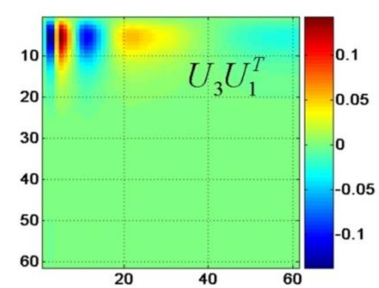
- second-order mean-field DCM



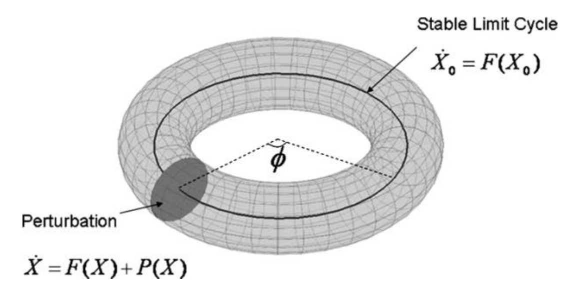
- DCM for steady-state responses



- DCM for induced responses



- DCM for phase coupling



Conclusion

planning a compatible DCM study

- Suitable experimental design:
  - any design that is suitable for a GLM
  - preferably multi-factorial (e.g. 2 x 2)
    - e.g. one factor that varies the driving (sensory) input
    - and one factor that varies the modulatory input

- Hypothesis and model:
  - define specific *a priori* hypothesis
  - which models are relevant to test this hypothesis?
  - check existence of effect on data features of interest
  - there exists formal methods for optimizing the experimental design for the ensuing bayesian model comparison [Daunizeau et al., PLoS Comp. Biol., 2011]

Many thanks to:

Karl J. Friston (UCL, London, UK)
Will D. Penny (UCL, London, UK)
Klaas E. Stephan (UZH, Zurich, Switzerland)
Stefan Kiebel (MPI, Leipzig, Germany)